FSA to DFSA Conversion

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http://www.cs.uiuc.edu/class/fa05/cs421/current/

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Elsa Gunter

Turning FSA into DFSA

• Need to treat all states reachable by a given path as the same
• States of DFSA are sets of states from original FSA
• The ε-closure of a set of state $S$, written here as $S(\epsilon)$, is the smallest set containing $S$
  - if $s \in S$ and $(s, \epsilon, t)$ is an edge labeled by $\epsilon$, then $t \in S$
• The $\alpha$-reachable states from $S$,
  $S(\alpha) = \{ t \mid (s, \alpha, t) \text{ is an edge for some } s \in S \}$

Turning FSA into DFSA

• Begin with $S = \{s_0\}(\epsilon)$, the ε-closure of the set containing only the start state; this is start state
• For each new state $S$ and letter $\alpha$, add state $S(\alpha)(\epsilon)$, the ε-closure of the $\alpha$-reachable states from $S$
• Add edge $(S, \alpha, S(\alpha)(\epsilon))$
• Done when all $S(\alpha)(\epsilon)$ are states that have already been added
• Final states: any set of states containing an original final state
• Problem: exponential in number of states

Example 1

New states: $\{A\}$
New edges: $(\{A\}, 0, \emptyset), (\{A\}, 1, \emptyset)$.
Example 1

New states: \{A\}, \{A, B\}
New edges: ((A), 0, \{A\}), ((A), 1, \{A, B\}),
((A, B), 0, \{A\}), ((A, B), 1, \{A, B\}),
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Example 2

New states: \{A\}(\epsilon) = \{A,B\},
New edges: ((A,B), 0, \epsilon), ((A,B), 1, \epsilon)

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New states: \{A,B\},
New edges: ((A,B), 0, \{A,B\}), ((A,B), 1, \{B\}(\epsilon))

New states: \{A,B\}, \{B\}(\epsilon) = \{B\}
New edges: ((A,B), 0, \{A,B\}), ((A,B), 1, \{B\})
Example 2

New states: \{A,B\}, \{B\}
New edges: \((A,B), 0, \{A,B\}\), \((A,B), 1, \{A,B\}\),
\((B), 0, \{A,B\}\), \((B), 1, \{B\}\)

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Turning Regular Expressions into NFSAs

- Compose NFAs for pieces of reg exp to get NFA for whole reg exp
- \(\epsilon\) goes to \((s_0, \{\}, \{s_0\}, s_0)\)
Turning Regular Expressions into NFSAs - letters

- $\varepsilon$ goes to $\{(s_0), \{\}, \{s_0\}, s_0\}$
- $\alpha$ goes to $\{(s_0, s_1), \{(s_0, \alpha, s_1)\}, \{s_1\}, s_0\}$

Turning Regular Expressions into NFSAs - Concatentation

- $(S, E, F, s_0)$ FSA for $\sigma$
- $(S', E', F', s_0')$ FSA for $\tau$
- Concatenation: $\sigma \tau$ goes to $(S \cup S', E \cup E' \cup \{(f, \varepsilon, s_0') | f \in F\}, F', s_0')$

Turning Regular Expressions into NFSAs - Choice

- $(S, E, F, s_0)$ FSA for $\sigma$
- $(S', E', F', s_0')$ FSA for $\tau$
- Choice: $\sigma \lor \tau$ goes to $(S \cup S' \cup \{s\}, E \cup E' \cup \{(s, \varepsilon, s_0), (s, \varepsilon, s_0')\}, F \cup F', s)$ where $s$ is a new state

Turning Regular Expressions into NFSAs - Kleene Star

- $(S, E, F, s_0)$ FSA for $\sigma$
- Choice: $\sigma^*$ goes to $(S \cup \{s'\}, E \cup \{(s', \varepsilon, s_0), (s', \varepsilon, s_0')\} \cup \{(f, \varepsilon, s') \mid f \in F\}, \{s'\}, s')$

Example: $(0 | 1)^* 1$

- $0$: $s_0 \xrightarrow{0} s_0$
- $1$: $s_2 \xrightarrow{1} s_5$
- $\varepsilon$: $s_1 \xrightarrow{\varepsilon} s_0 \xrightarrow{0} s_3 \xrightarrow{\varepsilon} s_0 \xrightarrow{1} s_4$

Example: $(0 | 1)^* 1$

- $(0 | 1)^*$: $s_0 \xrightarrow{\varepsilon} s_4 \xrightarrow{0} s_0 \xrightarrow{\varepsilon} s_3 \xrightarrow{1} s_1$
Example: $(0 \mid 1)^* \ 1$

- $(0 \mid 1)^* \ 1$: 

```
    s0
   / \  \
  ε   ε-
 /     |
s1  0   s2
 /     |
 s3  ε   s3
    ε   1
      ε   s5
```

s0, s1, s2, s3, s4, s5, s6