Quiz 7, Monday, October 30, 2006
ECE 598 AL
THE SPEECH CHAIN

Problem 1  (7 points)

Consider the following signal.

\[ x(t) = A \cos(880\pi t) + B \cos(1760\pi t + \pi/4) \]

(a) What is the fundamental frequency of this sound, in Hertz?

\[ \frac{880\pi}{2\pi} = 440 \text{ Hz} \]

(b) Suppose that the signal is sampled at a sampling frequency of \( F_s = 8800 \) samples/second. What is the fundamental period, in samples?

\[ \frac{8800 \text{ samples/sec}}{440 \text{ reps/sec}} = 20 \text{ samples/rep} \]

(c) Suppose this sound is filtered by a lowpass filter with a cutoff frequency of 450Hz. What will be the result?

\[ A \cos(880\pi t) \]

(d) Find the values of the following Fourier series coefficients, in terms of the constants \( A, B, \) and \( C, \) where \(< >\) denotes averaging over one fundamental period.

\[ X_1 = < x(t) \cos(880\pi t) > \]

\[ = \frac{A}{2} \]

\[ Y_1 = < x(t) \sin(880\pi t) > \]

\[ = 0 \]

\[ X_2 = < x(t) \cos(1760\pi t) > \]

\[ \cos(1760\pi t + \pi/4) = \frac{\sqrt{2}}{2} \cos(1760\pi t) - \frac{\sqrt{2}}{2} \sin(1760\pi t) \]

\[ X_2 = < x(t) \cos(1760\pi t) > = \frac{1}{2} B \frac{\sqrt{2}}{2} = \frac{B\sqrt{2}}{4} \]
Problem 2 (3 points)

The velocity of air coming through the glottis is periodic, with an amplitude that drops off roughly as $1/f^2$, i.e.

$$u_G(t) = e^{j\omega_0 t} + (1/4)e^{j2\omega_0 t} + (1/9)e^{j3\omega_0 t} + \ldots$$

where $\omega_0 = 2\pi F_0$, and $F_0 = 250Hz$ is the fundamental frequency. The recorded speech waveform is

$$X(\omega) = R(\omega)T(\omega)U_G(\omega)$$

where the vocal tract transfer function, for a single formant at $F_1 = 500Hz$ and $B_1 = 100Hz$, is given by

$$T(\omega) = \frac{(2\pi F_1)^2 + (\pi B_1)^2}{(j(\omega - 2\pi F_1) + (\pi B_1))(j(\omega + 2\pi F_1) + (\pi B_1))}$$

and the radiation characteristic at a distance of 1m is roughly

$$R(\omega) = j\omega/2\pi$$

Find the amplitude of the second harmonic of the recorded speech signal $x(t)$.

\[
\text{Second Harmonic} = \left| R(2\omega_0) T(2\omega_0) \left( \frac{1}{4} \right) e^{2j\omega_0 t} \right|
\]

\[
= \left| \left( \frac{2\pi}{2\pi} \right) \left( \frac{(100\pi)^2 + (100\pi)^2}{j(2\omega_0 - 100\pi) + 100\pi} \right) \right| \left( \frac{1}{4} \right)
\]

\[
= \left( 500j \right) \left( \frac{(1000)^2 + (100)^2}{100 \left( -j 2000 + 100 \right)} \right) \left( \frac{1}{4} \right) = \left| 500j \left( \frac{100^2 + 1^2}{-j 20 + 1} \right) \right| \left( \frac{1}{4} \right)
\]

\[
= 500 \left( \frac{101}{\sqrt{400 + 1}} \right) \left( \frac{1}{4} \right) = \frac{500}{4} \left( \frac{101}{\sqrt{401}} \right)
\]