Problem 6.1

During production of the vowel /a/, the pharynx is quite narrow (about 1cm$^2$), while the oral cavity is quite wide (about 8cm$^2$). Let the boundary between these two parts of the vocal tract be called $x = 0$.

(a) Draw a schematic picture of this situation.

(b) Pressure $p(x, t)$ (in Pascals) and volume velocity $u(x, t)$ (in liters/second) must be continuous across the boundary, i.e.

\begin{align*}
p(0-, t) &= p(0+, t) \\
u(0-, t) &= u(0+, t)
\end{align*}

Re-write Eqs. 1 and 2 in terms of the forward-going and backward-going waves, whose phasors are $p_{1+}$, $p_{1-}$, $p_{2+}$, and $p_{2-}$.

(c) Show that the outgoing waves from, $p_{2+}$ and $p_{1-}$, may be written in terms of the incoming waves, $p_{2-}$ and $p_{1+}$, and in terms of a reflection coefficient $\gamma$. Write $\gamma$ in terms of the front cavity and back cavity areas.

(d) Suppose that the glottis is a perfect source, i.e., regardless of what the backward-going wave $p_{1-}$ may be, the forward-going wave is always a perfect cosine $p_{1+} = 1$. Find the forward-going and backward-going waves in the front cavity, $p_{2+}$, and $p_{2-}$, as a function of the front cavity length $L_f$, and the reflection coefficient $\gamma$. Assume a zero-pressure termination at the lips.

(e) Find the air velocity at the lips, $v(L_f, \omega)$, as a function of $L_f$, $\omega$, and $\gamma$. Assume that $p_{1+} = 1$ at all frequencies.

(f) Plot $v(L_f, \omega)$ as a function of $\omega$.

Problem 6.2

Assume a perfectly decoupled back and front cavity, where $A_b \gg A_f$. Assume that $L_b + L_f = 17$cm. Calculate the first three formant frequencies for the following back cavity lengths: $L_b \in \{1, 3, 5, 7, 9, 11, 13, 15\}$cm. Remember to consider the Helmholtz resonance. Plot $F_1$, $F_2$, and $F_3$ (in Hertz) on the same axes, as a function of $L_b$. This plot is called a “nomogram;” it is considered by many to be a convenient summary of the relationship between vocal tract shape and vowel quality.