Problem 5.1

Each of the parts of this problem describes a cosine \( p(t) \), using either phasor, complex exponential, or real-valued time domain notation. Find the amplitude and phase of each cosine, and, in each case, sketch \( \Re \{ p(t) \} \) as a function of time.

(a) \[ p(t) = \cos(2000\pi t) + 0.5 \cos(2000\pi (t - 0.00025)) \]

(b) \[ p(t) = \cos(2000\pi t) + 1.5 \sin(2000\pi t) \]

(c) \[ p(t) = (1 + 5j) e^{j\omega t} \]

(d) \[ p = 2 - 2j \]

Problem 5.2

Consider a room that is exactly 4mx5m in size. Suppose that the floor is covered with a thick shag rug, and the ceiling is covered with sound-absorbing ceiling tiles, so that there are no echoes from either floor or ceiling. The walls, on the other hand, are brick, so the reflection coefficient of the walls is very nearly \( \gamma = 1 \) (meaning that any acoustic wave that hits the wall is immediately reflected in full).

An omnidirectional microphone is placed 1m from the east wall, and 1m from the south wall, in order to record everything that happens in the room (“omnidirectional” means that this microphone records sound equally well from all directions, rather than preferring sound from the front of the microphone). In the opposite corner (1m from the north wall, 1m from the west wall), a balloon of radius \( r_s = 20cm \) is popped at time \( t = 0 \), making a sound very similar to \( p(t) = \delta(t) \).
(a) Sketch the geometry of the room. Determine the time, $\tau_0$, at which the direct sound arrives at the microphone, and the scaling coefficient $a_0 = r_s/r_0$.

(b) Use the image source method to find the arrival times and scaling coefficients of any six additional echoes of your choice.

(c) Now pretend that the six echoes that you enumerated in part (b) are the only echoes; pretend that there are no later echoes. Write the impulse response, $h(t)$, as a series of seven scaled and shifted impulses (including the direct sound). Draw $h(t)$ as a function of time. You may symbolize the impulse $\delta(t - \tau_n)$ in any way that you like: as a tall thin rectangle, a tall thin triangle, or as a vertical arrow at time $\tau_n$ with the scaling constant written next to it in parentheses.

(d) Now suppose that, instead of a balloon popping, there is a person talking at the same position. The person is wearing a noise-cancelling close-talking headset microphone, so you have recordings of both $s(t)$ (the source waveform recorded by the headset microphone) and $x(t)$ (the reverberated waveform recorded by the omnidirectional microphone). Unfortunately, your recording of $x(t)$ was corrupted by a disk crash. Assuming that there are only six echoes in the room, how would you synthesize $x(t)$ from $s(t)$?

(e) Now suppose that, instead of a balloon popping or a person talking, there is a professional flautist at the same place (1m from north wall, 1m from west wall) producing beautiful pure tones of the form $p(t) = e^{j\omega t}$. As you know, under this condition, the recorded sound $x(t)$ is also a pure tone, i.e., $x(t) = Ae^{j(\omega t + \phi)}$. For the particular six echoes that you have been working with, calculate $A$ and $\phi$ at the following two frequencies. You will certainly want to use a calculator for this problem, but be sure to show enough work on your answer sheet so that I understand the method that you’ve used.

1. $f = 100\text{Hz}$.
2. $f = 1000\text{Hz}$.