UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498DJ Principles of Signal Analysis

Problem Set 4

Fall 2011

Assigned: 10/7/2011 **Due:** 10/14/2011

Problem 4.1

Consider the signal $x(t) = 3 + \cos(1000\pi t) + \sin(2000\pi t) + \cos(4000\pi t)$.

(a) Determine and list all of the analog frequencies in the signal $x(t)$.

(b) What is the lowest possible sampling frequency that would avoid aliasing?

(c) What is the corresponding Nyquist frequency for this sampling rate?

(d) For a sampling frequency of $F_s = 8000$Hz, find $x[n]$.

(e) Determine and list all of the frequencies ω, $-\pi < \omega \leq \pi$, present in the discrete-time signal $x[n]$.

(f) If we take a length-800 DFT of 800 samples of this signal, for which k values will the DFT samples $X[k]$ be nonzero, and to what analog frequencies do they correspond? Compute your answer by hand, then check it in matlab.

Problem 4.2

Suppose that we have a signal bandlimited to 8kHz. We want to digitally bandpass filter it to pass all signal components in the range $1000 \leq f \leq 2000$Hz, where $\Omega = 2\pi f$, and to eliminate all other frequencies.

(a) What is the minimum F_s necessary to avoid aliasing?

(b) For the sampling rate F_s that you chose in part (a), what are the corresponding bandpass edges, ω_l and ω_u, of the discrete-time filter $H_d(\omega)$?

(c) Sketch the frequency response $H_d(\omega)$ of the desired filter, for $0 \leq \omega \leq 2\pi$ (note the non-standard frequency range over whch I have asked you to sketch the frequency response!!)

(d) Suppose, instead, that the signal is sampled at $F_s = 44100$ samples/second (the sampling rate of an audio compact disc). What are the corresponding bandpass edges, ω_l and ω_u, of the discrete-time bandpass filter?

Problem 4.3

Assume that $x[n] = x_c(nT)$, where $1/T = 8000$ samples/second. For each of the following signals, find $x[n]$, $X_c(\Omega)$, and $X_d(\omega)$.

(a) $x_c(t) = \text{sinc}(2000\pi t)$ (sinc, not sin!)

(b) $x_c(t) = \cos(7000\pi t)$.

Problem 4.4

Define $y_0(t)$, $y_1(t)$, and $y_S(t)$ to be the piece-wise constant, piece-wise linear, and sinc interpolations of $x[n]$, respectively. For each of the following signals, sketch $y_0(t)$, $y_1(t)$, and $y_S(t)$ as functions of time. Sketch also $X_d(\omega)$, the DTFT of $x[n]$, and $Y_S(\Omega)$, the CTFT of $y_S(t)$.

(a) $x[n] = \delta[n]$

(b) $x[n] = \delta[n] - 0.5\delta[n+1] - 0.5\delta[n-1]$

Problem 4.5

Determine whether the following systems are linear, shift invariant, causal, and BIBO stable:

(a) $y[n] = x[n] - x[0]$

(b) $y[n] = x[n] - 1$

(c) $y[n] = x[n+1] + x[n] + x[n-1]$

(d) $y[n] = \sum_{m=0}^{\infty} x[n-m]$