Problem 3.1

Let’s start with the following signal:

\[ x_1[n] = \begin{cases} 
1 & 0 \leq n \leq 3 \\
0 & \text{otherwise} 
\end{cases} \]

(a) The discrete-time Fourier transform of \( x[n] \) can be written as

\[ X_1(\omega) = e^{-j\omega L} A(\omega), \]

for some real-valued \( L \) and real-valued \( A(\omega) \). Find \( L \) and \( A(\omega) \).

(b) Find the form of the 8-point DFT, \( X_1[k] \), for \( 0 \leq k \leq 7 \), by just sampling your answer to part (a) at the right values of \( \omega \).

(c) Evaluate your answer to part (b) in order to find the actual numerical values of \( X_1[k] \), for \( 0 \leq k \leq 7 \).

(d) Show that your answer to part (c) satisfies the conjugate-symmetry property.

Problem 3.2

Consider the following two signals:

\[ x_2[n] = \begin{cases} 
1 & 0 \leq n \leq 6 \\
0 & \text{otherwise} 
\end{cases} \]

\[ y_2[n] = \begin{cases} 
1 & -3 \leq n \leq 3 \\
0 & \text{otherwise} 
\end{cases} \]

Find the DTFT of either of these two signals (hint: you can just apply your answer from problem 1, but with different constants). Then use the time-shift property to find the DTFT of the other signal.

Problem 3.3

Consider the following two signals:

\[ x_3[n] = \begin{cases} 
1 & -7 \leq n \leq 7 \\
0 & \text{otherwise} 
\end{cases} \]

\[ y_3[n] = \begin{cases} 
\cos \left( \frac{5\pi n}{7} \right) & -3 \leq n \leq 3 \\
0 & \text{otherwise} 
\end{cases} \]
Find the DTFT of either of these two signals (hint: you can just apply your answer from problem 2, but with different constants). Then use the modulation property to find the DTFT of the other signal (hint: use Euler’s relations to express the cosine in terms of complex exponentials, then apply the modulation property). Sketch $Y_3(\omega)$ for $-\pi \leq \omega \leq \pi$.

**Problem 3.4**

Show that the inverse DTFT of $X_4(\omega) = 2\pi \delta(\omega - \pi)$ is $x_4[n] = (-1)^n$, i.e., the sequence

$$x_4[n] = \{1, -1, 1, -1, 1, \ldots\}$$