Problem 7.1

A periodic continuous-time signal has the Fourier series
\[ x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T_0} \]

Suppose that \( T_0 = 0.01 \)s. Suppose that \( x(t) \) is lowpass filtered by an ideal anti-aliasing filter with a cutoff of 5kHz, then sampled at \( F_s = 10 \)kHz to create \( x[n] \). \( x[n] \) is then passed through a 50-sample averager to create \( y[n] \):
\[ y[n] = \frac{1}{50} \sum_{m=0}^{49} x[n-m] \]

The signal \( y[n] \) is sent through an ideal D/A with the same sampling frequency, \( F_s = 10 \)kHz, to create the signal \( y(t) \), which can be written as
\[ x(t) = \sum_{k=-\infty}^{\infty} Y_k e^{j2\pi kt/T_0} \]

(a) \( Y_k = 0 \) for \( |k| \geq 50 \) because of the anti-aliasing filter, and for \( k = 2\ell \) because of the discrete-time averaging.

(b) The frequency of the first null is \( \omega_c = 2\pi/50 \) radians/second. In Hertz, this is
\[ \left[ \frac{2\pi \text{ radians}}{50 \text{ sample}} \right] \times \left[ \frac{10,000 \text{ samples}}{\text{second}} \right] \times \left[ \frac{1 \text{ cycles}}{2\pi \text{ radian}} \right] = 200 \text{ cycles/second} \]

(c)
\[ |Y_k| = \begin{cases} |X_k| & k = 0 \\ 0 & |k| \geq 50 \\ 0 & k = 2\ell, \text{ integer } \ell \\ \frac{\sin(\pi k/2)}{50\sin(\pi k/50)} |X_k| & \text{otherwise} \end{cases} \]

Problem 7.2
A periodic continuous-time signal has the Fourier series

\[ x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T_0} \]

Suppose that \( T_0 = 0.01 \text{s} \). Suppose that \( x(t) \) is lowpass filtered by an ideal anti-aliasing filter with a cutoff of 4kHz, then sampled at \( F_s = 8\text{kHz} \) to create \( x[n] \). \( x[n] \) is then passed through a 40-sample averager to create \( y[n] \):

\[ y[n] = \frac{1}{40} \sum_{m=0}^{39} x[n - m] \]

The signal \( y[n] \) is sent through an ideal D/A with the same sampling frequency, \( F_s = 8\text{kHz} \), to create the signal \( y(t) \), which can be written as

\[ x(t) = \sum_{k=-\infty}^{\infty} Y_k e^{j2\pi kt/T_0} \]

(a) \( Y_k = 0 \) for \( |k| \geq 40 \) because of the anti-aliasing filter, and for \( k = 2\ell \) because of the discrete-time averaging.

(b) The frequency of the first null is \( \omega_c = 2\pi/40 \text{ radians/second} \). In Hertz, this is

\[ \left[ \frac{2\pi \text{ radians}}{40 \text{ sample}} \right] \times \left[ 8000 \text{ samples/second} \right] \times \left[ \frac{1 \text{ cycles}}{2\pi \text{ radian}} \right] = 200 \text{ cycles/second} \]

(c)

\[ |Y_k| = \begin{cases} 
|X_k| & k = 0 \\
0 & |k| \geq 40 \\
0 & k = 2\ell, \text{ integer } \ell \\
\left| \frac{\sin(\pi k/2)}{40 \sin(\pi k/40)} X_k \right| & \text{otherwise}
\end{cases} \]